**A Model for Analysis of Bond Valuation**

**Abstract**

A bond is a type of asset in which a government or a company issues these securities as a long – term debt to borrow money from institutional investors (Banks) or public sector. Each bond has a time to maturity which is usually 5, 10, and 20 or 30 years. Initial value of bond is named par value or face value equal to $1000 with a coupon interest rate on the bond which is the percentage of par value and it will be paid annually or semiannually (two times in one year). In fact, the government or the company is committed regularly and continuously to pay these payments and also repayment of initial value (par value) at maturity time.

The purpose of this article is to present a model for analysis of bond value where there are seven independent variables including current bond price (bond value), YTM, coupon rate, purchased year, purchased month, purchased day and time period (n). This model simultaneously solves an equation with three independent variables accompanied by generating maximum and minimum of this function for given domain and range and also non simultaneously analyzes seven independent variables. One of the most crucial applications of this model is to obtain YTM (return rate) for current price equal to bond value without using any trial and error.

A coupon interest rate always stay the constant while the purchasers of bonds strongly look at and compare it with premium risk of market which is named the return rate or required rate on the capital or yield to maturity (YTM).

**Introduction**

Since the late 1970s, term-structure (TS) theory has evolved from qualitative propositions about shapes of interest-rate curves to very specific, non-linear models that price both bonds and derivatives. Following Sercu and Wu (1997), our test of eight such models center on the question how much money can be made by trading on the deviations between observed bond prices and values proposed by bond-pricing models. Sercu and Wu (SW) report that such trading generates abnormal returns. One improvement, in the present paper, is that we extend their work to a longitudinally as well as cross-sectionally larger sample and add more models, especially two-factor models. But the more interesting contributions, we think, concern the methodology and the conclusions. First, we come up with just one benchmark, and it is not biased and is more efficient. SW, in contrast, use three benchmarks of (then) untested validity and efficiency. But when in a trial run we applied the SW trading rule to a-select portfolios (like buying short-term bonds only, or long-term bonds only), we found that some of these naive buy-and-hold strategies seemed to provide abnormal returns too, by SW standards. If a-select portfolios already seem to provide abnormal returns, then the finding that a selective trading rule is profitable sounds less impressive: the cause may just be a flawed benchmark for the normal return. So this prompted us to look for a new benchmark-return strategy that avoids such biases and minimizes noise.

A coupon bond promisers to pay whoever owns the bond certain interest payment (coupon) at specified dates in the future plus the face value (1,000 unless otherwise stated) of the bond at maturity.

To price the bond, we must figure out the present value of all the future payments.

What information we need:

The coupon rate

The yield to maturity (sometimes called the current interest rate, market interest rate)

The time to maturity (life of the bond)

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## MODELLING

The models we work with are, in order of complexity, (*i*) the cubic spline; (*ii*) two seminal onefactor models, (*iii*) four two-factor models. Most of these are widely known, but to identify the parameter estimates presented below we nevertheless need to agree on a notation. Thus, the key factor processes or equations are presented below.

*The Vasicek model.* Vasicek (1977) assumes a mean-reverting Gaussian process for the instantaneous interest rate,

*dr*(*t*) = *α*(*β* − *r*(*t*))*dt* + *σdW*(*t*)*,* (1)

where *α >* 0 is the mean reversion parameter, *β* the unconditional mean of r(t), *σ* the volatility of the spot rate, and *W*(*t*) a standard Brownian motion. The price of risk is assumed to be constant.

*The Cox-Ingersoll-Ross Model.* The second model, by Cox, Ingersoll and Ross (1985), is generalequilibrium in nature. It assumes log-utility investors facing a mean-reverting squareroot process for output, and from these derives a mean-reverting square-root process for the instantaneous rate and an endogenous price of risk. The process for *r* is

q *dr*(*t*) = *α*(*β* − *r*(*t*))*dt* + *σ r*(*t*)*dW*(*t*) *,* (2)

where *α >* 0 is the mean reversion parameter, *β* the unconditional mean of *r*(*t*), *σ* a measure of volatility of the spot rate, and *W*(*t*) a standard Brownian motion.

*The Richard Model.* Starting from the Fisher equation, Richard (1978) assumes that the instantaneous real interest rate (*R*) and the expected inflation rate (*π*) each follow a mean-reverting squareroot process:

√

*dR*(*t*) = *a*(*R*∗ − *R*)*dt* + *σR R dZR*(*t*)*,* and (3) *dπ*(*t*) = *c*(*π*∗ − *π*)*dt* + *σπ*√*π dZπ*(*t*)*.* (4)

The correlation between *ZR* and *Zπ* is assumed to be zero. Actual inflation is expected inflation plus noise, and the nominal rate is the real rate plus expected inflation:

|  |  |  |  |
| --- | --- | --- | --- |
| *dP*(*t*)*/P*(*t*) | = | *π*(*t*)*dt* + *σP* (*π,R*)*dZP* (*t*)*,* and | (5) |
| *r*(*t*) | = | *R*(*T*) + *π*(*t*)(1 − *σP*2 )*.* | (6) |

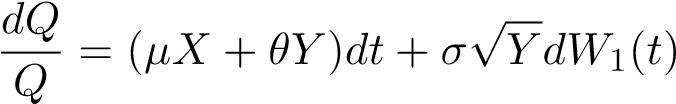
*The Longstaff and Schwartz model.* Longstaff and Schwartz (1992) develop a two-factor general equilibrium model of the term structure that builds upon CIR. They take the short-term interest rate and the instantaneous variance of the short-term interest rate as the two driving factors. The mathematical structure is very similar to Richards’, though. Initially, Longstaff and Schwartz assume two unobservable state variables, *X* and *Y* , which follow squareroot processes,

√ *dX* = (*a* − *bX*)*dt* + *c XdW*2(*t*)*,* and (7)

√

*dY* = (*d* − *eY* )*dt* + *f Y dW*3(*t*)*,* (8)

and which affect expected returns on investment as follows:

*,* (9)

where *W*2 is assumed to be uncorrelated with *W*1 and *W*3. Assuming log utility, expected growth in marginal utility—the instantaneous interest rate—is expected output minus variance of output. Thus,

*r*(*t*) = *αx* + *βy* (10)

where *α* = *µc*2, *β* = (*θ* − *σ*2)*f*2, *x* = *X/c*2, *y* = *Y/f*2, *γ* = *a/c*, *δ* = *b*, *η* = *d/f*2 and *ξ* = *e*. The

variance of changes in the short-term interest rate is

*V* (*t*) = *α*2*x* + *β*2*y.* (11)

*The Balduzzi, Das, Foresi and Sundaram model.* Balduzzi, Das, Foresi and Sundaram (2000) develop a two-factor model where the first factor *r* is the short rate and the second factor, *θ*, is the mean level of the short rate (in the sense of the long-run level to which the rate is attracted, everything else being the same). The short rate follows the same process as in the Vasicek setting,

|  |  |
| --- | --- |
| *dr*(*t*) = *κ*(*θ* − *r*)*dt* + *σdW*1(*t*)*.*  except that it is attracted not to a constant mean but to a moving target, with | (12) |
| *dθ*(*t*) = (*a* + *bθ*)*dt* + *ηdW*2(*t*)*,* | (13) |

with *a*, *b* and *η* constants. The two processes can be correlated: *dW*1*dW*2 = *ρdt*. The prices of risk are assumed to be constant.

*The Baz and Das model.* Baz and Das (1996) extend the Vasicek model by adding a Poisson jump process N(t) with intensity rate *λ*. The process for the short-term rate in the extended Vasicek jump-diffusion process then becomes:

*dr*(*t*) = *α*(*β* − *r*(*t*))*dt* + *σdW*(*t*) + *JdN*(*t*)*.* (14)

with *α* the mean reversion coefficient, *β* the long-term mean of the short interest rate, and *σ* the instantaneous volatility. The intensity of the jump is defined by *J*, which is assumed to be a normal variable with mean *θ* and a standard deviation of *δ*. This one-factor model jump-diffusion model can be easily extended when one assumes two orthogonal factors. To that end two similar processes can be defined:

|  |  |  |  |
| --- | --- | --- | --- |
| *dy*1(*t*) | = | *α*1 (*β*1 − *y*1(*t*))*dt* + *σ*1*dW*1(*t*) + *J*1*dN*1(*t*) | (15) |
| *dy*2(*t*) | = | *α*2 (*β*2 − *y*2(*t*))*dt* + *σ*2*dW*2(*t*) + *J*2*dN*2(*t*)*,* | (16) |
| *r*(*t*) | = | *y*1(*t*) + *y*2(*t*) | (17) |

where *dy*1(*t*) and *dy*2(*t*) are independent.

*The Cubic Spline.* McCulloch (1975) uses the cubic spline to curve-fit the TS. The price of a discount bond with remaining life *τ* is then given by

*K*

*P*(*τ*) = *a*1*τ* + *a*2*τ*2 + *a*3*τ*3 + X*di* [*max*(*τ* − *kj,*0)]3 (18)

*j*=1

where *ki* are the *K* knot points or knots. These divide the maturity range into *K* + 1 distinct sections, within each of which the TS follows a cubic and where the cubics smoothly join at the knots. The choice of the number of knots and their values is rather arbitrary. For comparability with Sercu and Wu, we set two knots, at 2 and 7 years. The parameters *a*1, *a*2, *a*3, *d*1 and *d*2 can be estimated by an ordinary linear regression.

This finishes our presentation of the models and their notation; the data to which these models are taken come next.

**RESULTS AND DISCUSSION**

**General Linear Model**

|  |  |  |
| --- | --- | --- |
| **Notes** | | |
| Output Created | | 30-JUL-2021 18:23:18 |
| Comments | |  |
| Input | Active Dataset | DataSet3 |
| Filter | <none> |
| Weight | <none> |
| Split File | <none> |
| N of Rows in Working Data File | 1927 |
| Missing Value Handling | Definition of Missing | User-defined missing values are treated as missing. |
| Cases Used | Statistics are based on all cases with valid data for all variables in the model. |
| Syntax | | GLM funding1 funding2 funding3 funding4 funding5 funding6 BY reason1 reason2 reason3 reason4  reason5 reason6 WITH ratings  /REGWGT=tender  /METHOD=SSTYPE(3)  /INTERCEPT=INCLUDE  /CRITERIA=ALPHA(.05)  /DESIGN=reason1\*reason2\*reason3\*reason4. |
| Resources | Processor Time | 00:00:00.03 |
| Elapsed Time | 00:00:00.10 |

|  |
| --- |
| **Warnings** |
| The following factors or covariates are not used in the model: reason5, reason6, ratings |

|  |  |  |
| --- | --- | --- |
| **Between-Subjects Factors** | | |
|  | | N |
| reason1 | 0 | 125 |
| 1 | 360 |
| reason2 | 0 | 456 |
| 1 | 29 |
| reason3 | 0 | 482 |
| 1 | 3 |
| reason4 | 0 | 479 |
| 1 | 6 |
| reason5 | 0 | 483 |
| 1 | 2 |
| reason6 | 0 | 417 |
| 1 | 68 |

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| **Multivariate Testsa,b** | | | | | | |
| Effect | | Value | F | Hypothesis df | Error df | Sig. |
| Intercept | Pillai's Trace | .515 | 101.095c | 5.000 | 476.000 | .000 |
| Wilks' Lambda | .485 | 101.095c | 5.000 | 476.000 | .000 |
| Hotelling's Trace | 1.062 | 101.095c | 5.000 | 476.000 | .000 |
| Roy's Largest Root | 1.062 | 101.095c | 5.000 | 476.000 | .000 |
| reason1 \* reason2 \* reason3 \* reason4 | Pillai's Trace | 1.225 | 42.291 | 20.000 | 1916.000 | .000 |
| Wilks' Lambda | .182 | 52.968 | 20.000 | 1579.663 | .000 |
| Hotelling's Trace | 2.518 | 59.740 | 20.000 | 1898.000 | .000 |
| Roy's Largest Root | 1.469 | 140.776d | 5.000 | 479.000 | .000 |
| a. Design: Intercept + reason1 \* reason2 \* reason3 \* reason4 | | | | | | |
| b. Weighted Least Squares Regression - Weighted by tender | | | | | | |
| c. Exact statistic | | | | | | |
| d. The statistic is an upper bound on F that yields a lower bound on the significance level. | | | | | | |

The factors of funding state that most of the bonds are funded by equity which can be derived from the results.

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| **Tests of Between-Subjects Effectsa** | | | | | | |
| Source | Dependent Variable | Type III Sum of Squares | df | Mean Square | F | Sig. |
| Corrected Model | funding1 | 8.250b | 4 | 2.063 | 9.207 | .000 |
| funding2 | 7.899c | 4 | 1.975 | 48.940 | .000 |
| funding3 | 19.491d | 4 | 4.873 | 26.170 | .000 |
| funding4 | 4.173e | 4 | 1.043 | 35.563 | .000 |
| funding5 | 16.639f | 4 | 4.160 | 91.909 | .000 |
| funding6 | .000g | 4 | .000 | . | . |
| Intercept | funding1 | 2.200 | 1 | 2.200 | 9.822 | .002 |
| funding2 | .788 | 1 | .788 | 19.529 | .000 |
| funding3 | .383 | 1 | .383 | 2.056 | .152 |
| funding4 | 1.937 | 1 | 1.937 | 66.017 | .000 |
| funding5 | .425 | 1 | .425 | 9.384 | .002 |
| funding6 | .000 | 1 | .000 | . | . |
| reason1 \* reason2 \* reason3 \* reason4 | funding1 | 8.250 | 4 | 2.063 | 9.207 | .000 |
| funding2 | 7.899 | 4 | 1.975 | 48.940 | .000 |
| funding3 | 19.491 | 4 | 4.873 | 26.170 | .000 |
| funding4 | 4.173 | 4 | 1.043 | 35.563 | .000 |
| funding5 | 16.639 | 4 | 4.160 | 91.909 | .000 |
| funding6 | .000 | 4 | .000 | . | . |
| Error | funding1 | 107.531 | 480 | .224 |  |  |
| funding2 | 19.367 | 480 | .040 |  |  |
| funding3 | 89.375 | 480 | .186 |  |  |
| funding4 | 14.082 | 480 | .029 |  |  |
| funding5 | 21.724 | 480 | .045 |  |  |
| funding6 | .000 | 480 | .000 |  |  |
| Total | funding1 | 191.000 | 485 |  |  |  |
| funding2 | 29.000 | 485 |  |  |  |
| funding3 | 165.000 | 485 |  |  |  |
| funding4 | 19.000 | 485 |  |  |  |
| funding5 | 42.000 | 485 |  |  |  |
| funding6 | .000 | 485 |  |  |  |
| Corrected Total | funding1 | 115.781 | 484 |  |  |  |
| funding2 | 27.266 | 484 |  |  |  |
| funding3 | 108.866 | 484 |  |  |  |
| funding4 | 18.256 | 484 |  |  |  |
| funding5 | 38.363 | 484 |  |  |  |
| funding6 | .000 | 484 |  |  |  |
| a. Weighted Least Squares Regression - Weighted by tender | | | | | | |
| b. R Squared = .071 (Adjusted R Squared = .064) | | | | | | |
| c. R Squared = .290 (Adjusted R Squared = .284) | | | | | | |
| d. R Squared = .179 (Adjusted R Squared = .172) | | | | | | |
| e. R Squared = .229 (Adjusted R Squared = .222) | | | | | | |
| f. R Squared = .434 (Adjusted R Squared = .429) | | | | | | |
| g. R Squared = . (Adjusted R Squared = .) | | | | | | |

SUMMARY

The factors of funding state that most of the bonds are funded by equity which can be derived from the resul

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